

对二次型图形的探讨

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【摘要】针对二次型的综合性和抽象性,利用二次型的几何意义首先阐述二次型的标准形所表示的图形,进而得出二次型的本质,建立二次型的直观概念。

【关键词】二次型;几何意义;标准形;本质

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二次型在《高等代数》中占有重要地位,应用也非常广泛^[1,2],但在教学过程中经常会遇到学生提出这样的问题:二次型到底是什么?它有什么几何意义?二次型与其标准形有什么区别和联系?本文以二元、三元二次型为例就这些问题进行一分析和讨论。

1 二元二次型的几何意义

设有二元二次型的标准形 $f(x, y) = d_1x^2 + d_2y^2$

(1)若正惯性指数为2,即 $d_1 > 0, d_2 > 0$,则 $d_1x^2 + d_2y^2 = c (c > 0)$ 可化为 $\frac{x^2}{(\sqrt{c/d_1})^2} + \frac{y^2}{(\sqrt{c/d_2})^2} = 1$,它代表椭圆。

(2)若正惯性指数为1,负惯性指数为0,不妨设 $d_1 > 0, d_2 = 0$,则 $d_1x^2 + d_2y^2 = c (c \geq 0)$,当 $c > 0, x = \pm \sqrt{c/d_1}$,它代表两平行直线;当 $c = 0, x = 0$,它代表y轴。

(3)若正惯性指数为1,负惯性指数为1,不妨设 $d_1 > 0, d_2 < 0$,则 $d_1x^2 + d_2y^2 = c$,当 $c > 0, \frac{x^2}{(\sqrt{c/d_1})^2} - \frac{y^2}{(\sqrt{-c/d_2})^2} = 1$ 它代表双曲线;当 $c = 0, y = \pm \sqrt{-d_1/d_2}x$,它代表两相交直线;当 $c < 0, \frac{y^2}{(\sqrt{c/d_2})^2} - \frac{x^2}{(\sqrt{-c/d_1})^2} = 1$,它代表双曲线。

(4)若正惯性指数为0,负惯性指数为1,不妨设 $d_1 = 0, d_2 < 0$,则 $d_1x^2 + d_2y^2 = c (c \leq 0)$,当 $c < 0, y = \pm \sqrt{c/d_2}$ 它代表两平行直线;当 $c = 0, y = 0$,它代表x轴。

(5)若负惯性指数为2,即 $d_1 < 0, d_2 < 0$,则 $d_1x^2 + d_2y^2 = c (c < 0)$ 可化为 $\frac{x^2}{(\sqrt{c/d_1})^2} + \frac{y^2}{(\sqrt{c/d_2})^2} = 1$,它代表椭圆。

由以上讨论可知,二元二次型的标准形在本质上是xoy平面(二维空间)上的图形,而任何一个二次型都可以通过非退化的线性替换(坐标旋转)化为标准形^[3],所以任何一个二元二次型都代表xoy平面(二维空间)上的图形。

2 三元二次型的几何意义

设有三元二次型的标准形 $f(x, y, z) = d_1x^2 + d_2y^2 +$

d_3z^2

(1)若正惯性指数为3,即 $d_1 > 0, d_2 > 0, d_3 > 0$,则 $d_1x^2 + d_2y^2 + d_3z^2 = c (c > 0)$ 可化为

$$\frac{x^2}{(\sqrt{c/d_1})^2} + \frac{y^2}{(\sqrt{c/d_2})^2} + \frac{z^2}{(\sqrt{c/d_3})^2} = 1, \text{它代表椭球面。}$$

(2)若正惯性指数为2,负惯性指数为0,不妨设 $d_1 > 0, d_2 > 0, d_3 = 0$,则 $d_1x^2 + d_2y^2 + d_3z^2 = c (c \geq 0)$,当 $c > 0$ 时,则 $d_1x^2 + d_2y^2 = c$,即 $\frac{x^2}{(\sqrt{c/d_1})^2} + \frac{y^2}{(\sqrt{c/d_2})^2} = 1$,它代表母线平行z轴的圆柱面;当 $c = 0$ 时,它代表z轴。

(3)若正惯性指数为2,负惯性指数为1,不妨设 $d_1 > 0, d_2 > 0, d_3 < 0$,则 $d_1x^2 + d_2y^2 + d_3z^2 = c$,当 $c > 0$ 时, $\frac{x^2}{(\sqrt{c/d_1})^2} + \frac{y^2}{(\sqrt{c/d_2})^2} - \frac{z^2}{(\sqrt{-c/d_3})^2} = 1$,它代表单叶双曲面;当 $c = 0$ 时, $z^2 = -\frac{d_1}{d_3}x^2 - \frac{d_2}{d_3}y^2$,它代表椭圆锥面;当 $c < 0$ 时, $-\frac{x^2}{(\sqrt{-c/d_1})^2} - \frac{y^2}{(\sqrt{-c/d_2})^2} + \frac{z^2}{(\sqrt{c/d_3})^2} = 1$,它代表双叶双曲面。

(4)若正惯性指数为1,负惯性指数为0,不妨设 $d_1 > 0, d_2 = d_3 = 0$,则 $d_1x^2 + d_2y^2 + d_3z^2 = c (c \geq 0)$,当 $c > 0, x = \pm \sqrt{c/d_1}$,它代表两平行平面;当 $c = 0, x = 0$,它代表yoz平面。

(5)若正惯性指数为1,负惯性指数为1,不妨设 $d_1 > 0, d_2 < 0, d_3 = 0$,则 $d_1x^2 + d_2y^2 + d_3z^2 = c$,当 $c > 0$ 时, $\frac{x^2}{(\sqrt{c/d_1})^2} - \frac{y^2}{(\sqrt{-c/d_2})^2} = 1$,它代表母线平行z轴的双曲柱面;当 $c = 0$ 时, $y = \pm \sqrt{-d_1/d_2}x$,它代表两相交平面;当 $c < 0$ 时, $-\frac{x^2}{(\sqrt{-c/d_1})^2} + \frac{y^2}{(\sqrt{-c/d_2})^2} = 1$,它代表母线平行z轴的双曲柱面。

(6)若正惯性指数为1,负惯性指数为2,讨论同(3)。

(7)若正惯性指数为0,负惯性指数为1,讨论同(4)。

(8)若正惯性指数为0,负惯性指数为2,讨论同

(2)。(三维空间)上的图形。

(9)若正惯性指数为0,负惯性指数为3,讨论同

3 结束语

(1)。
由以上讨论,可建立二次型的直观概念,任何一个n元二次型都代表n维空间上的图形。

综上,任何一个三元二次型都代表oxyz空间

注释及参考文献:

[1]王海东.正定二次型的刻划定理及其程序[J].长春大学学报,2006,16(3):27-30.

[2]牛滨华,孙晟,孙春岩,等.地震波的场方程矩阵和能量的正定二次型及其意义[J].地球物理学进展,2007,22(2):353-358.

[3]北京大学数学系几何与代数教研室前代数小组.高等代数(第三版)[M].北京:高等教育出版社,2003:210.

Discussion on the Graph of Quadratic Forms

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Abstract: According to the comprehensiveness and abstractness of quadratic forms, we first elaborate graphics of the canonical forms of quadratic forms by the geometric significance, then come to the essence of quadratic forms, and establish the visual concepts of quadratic forms.

Key words: Quadratic forms; Geometric significance; Canonical forms; Essence

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seeding stage, squaring stage, blooming stage, flowering and fruiting period and mature period during the growth of the golden buckwheat. Anglicizing by synthesis, the best harvest time of golden buckwheat is the period between the seeds maturation and the first frost, because during this time, the golden buckwheat is with the largest accumulation of dry matter and active constituent.

Key words: Golden buckwheat; Cultivation; Yield; Active constituent

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emulsion vaccine of H5 and H9 were used in the test. The results showed that the H5 and H9 maternal antibody level were highest in 1-day-age chicken and cut down gradually. The H5 and H9 maternal antibody in Broiler chicken were respectively decreased to $4.2\log_2$ and $4.3\log_2$ after 11-day-age and that of Luoman chicken were decreased to $4.2\log_2$ and $4.8\log_2$ after 28-day-age. Broiler chicken and Luoman chicken could be respectively first immunized in 11-day-age and 7-day-age. The immune program used in the test could usefully prevent Avian Influenza in chicken.

Key words: Avian influenza; Maternal antibody; HI antibody; Immunization; Vaccine