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粗双枝模糊集的扩展原理

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【摘 要】利用粗双枝模糊集的表现定理,给出粗双枝模糊集的扩展原理,为知识的表示提供了一种方法。

【关键词】粗双枝模糊集;表现定理;扩展原理

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1 引言

在Zadeh L A提出模糊集理论[1]和Pawlak Z提出的粗糙集理论[2]的基础上,Nanda和Majumdar于1992年给出了模糊粗糙集[3]的概念,模糊粗糙集的概念丰富了对信息系统中不完善、不准确知识的描述、处理,利用了模糊集和粗糙集在处理不完善,不准确性知识中的长处。在Zadeh L A模糊集的基础上,以工程决策,工程控制系统为实际背景,史开泉[4]提出双枝模糊集,刘若慧等[5]给出了粗双枝模糊集的概念,随后一些学者[6,7]对粗双枝模糊集的分解定理、表现定理作出讨论,在此基础上,本文给出粗双枝模糊集的扩展原理。

约定:BBFS(X)表示X上的双枝模糊集,RBBFS(X)为近似空间(X,R_1)上的粗双枝模糊集全体,RBBFS(Y)为近似空间(Y,R_2)上的粗双枝模糊集全体。

2 粗双枝模糊集的扩展原理

下面一些符号及运算法则参见刘若慧等研究[7]。

定义1　设映射f:X→Y,x|→f(x),f可以诱导出一个从RBBFS(X)到RBBFS(Y)的映射及一个从RBBFS(Y)到RBBFS(X)的映射

$$f:RBBFS(X)\to RBBFS(Y):(\underline{A},\overline{A})\mapsto f(\underline{A},\overline{A})$$

$$f^{-1}:RBBFS(Y)\to RBBFS(X):(\underline{B},\overline{B})\mapsto f^{-1}(\underline{B},\overline{B})$$

$f(\underline{A},\overline{A})$与$f^{-1}(\underline{B},\overline{B})$的隶属函数分别定义为:

$$f(\underline{A},\overline{A})(y)=(\bigvee_{f(x)=y}\underline{A}(x),\bigvee_{f(x)=y}\overline{A}(x))$$

$$f^{-1}(\underline{B},\overline{B})(x)=(\underline{B}(f(x)),\overline{B}(f(x)))$$

称$f(\underline{A},\overline{A})$为$(\underline{A},\overline{A})$的像,称$f^{-1}(\underline{B},\overline{B})$为$(\underline{B},\overline{B})$的逆像。

由定义1可得$\underline{f(\underline{A},\overline{A})}=f(\underline{A}),\overline{f(\underline{A},\overline{A})}=f(\overline{A})$

定理1设映射f:X→Y,x|→y

1)若$(\underline{A},\overline{A})\in$RBBFS(X),则$f(\underline{A},\overline{A})=(\bigcup_{\lambda\in[-1,1]}\lambda f(\underline{A}_\lambda),\bigcup_{\lambda\in[-1,1]}\lambda f(\overline{A}_\lambda))$

2)若$(\underline{B},\overline{B})\in$RBBFS(Y),则$f^{-1}(\underline{B},\overline{B})=(\bigcup_{\lambda\in[-1,1]}\lambda f^{-1}(\underline{B}_\lambda),\bigcup_{\lambda\in[-1,1]}\lambda f^{-1}(\overline{B}_\lambda))$

证明:1)a)设$\lambda\in[-1,0]$, $\forall y\in Y^-\cup Y^0$

$$\begin{aligned}
&(\bigcup_{\lambda\in[-1,0]}\lambda f(\underline{A}_\lambda),\bigcup_{\lambda\in[-1,0]}\lambda f(\overline{A}_\lambda))(y)\\
&=((\bigcup_{\lambda\in[-1,0]}\lambda f(\underline{A}_\lambda))(y),(\bigcup_{\lambda\in[-1,0]}\lambda f(\overline{A}_\lambda))(y))\\
&=(\bigvee_{\lambda\in[-1,0]}(\lambda\wedge f(\underline{A}_\lambda)(y)),\bigvee_{\lambda\in[-1,0]}(\lambda\wedge f(\overline{A}_\lambda)(y)))\\
&=(\bigvee_{\lambda\in[-1,0]}(\lambda\wedge(\bigvee_{f(x)=y}\underline{A}_\lambda(x))),\bigvee_{\lambda\in[-1,0]}(\lambda\wedge(\bigvee_{f(x)=y}\overline{A}_\lambda(x))))\\
&=(\bigvee_{\lambda\in[-1,0]}(\bigvee_{f(x)=y}(\lambda\wedge\underline{A}_\lambda(x))),\bigvee_{\lambda\in[-1,0]}(\bigvee_{f(x)=y}(\lambda\wedge\overline{A}_\lambda(x))))\\
&=(\bigvee_{f(x)=y}(\bigvee_{\lambda\in[-1,0]}(\lambda\wedge\underline{A}_\lambda(x))),\bigvee_{f(x)=y}(\bigvee_{\lambda\in[-1,0]}(\lambda\wedge\overline{A}_\lambda(x))))\\
&=(\bigvee_{f(x)=y}\underline{A}(x),\bigvee_{f(x)=y}\overline{A}(x))
\end{aligned}$$

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b)设 $\lambda\in[0,1]$, $\forall y\in Y^0\cup Y^+$,可类似证明。

2)a)设 $\lambda\in[-1,0]$, $\forall x\in X^-\cup X^0$

$$(\bigcup_{\lambda\in[-1,0]}\lambda f^{-1}(\underline{B}_\lambda),\bigcup_{\lambda\in[-1,0]}\lambda f^{-1}(\overline{B}_\lambda))(x)$$
$$=((\bigcup_{\lambda\in[-1,0]}\lambda f^{-1}(\underline{B}_\lambda))(x),(\bigcup_{\lambda\in[-1,0]}\lambda f^{-1}(\overline{B}_\lambda))(x))$$
$$=((\bigvee_{\lambda\in[-1,0]}\lambda f^{-1}(\underline{B}_\lambda))(x),(\bigcup_{\lambda\in[-1,0]}\lambda f^{-1}(\overline{B}_\lambda))(x))$$
$$=(\bigvee_{\lambda\in[-1,0]}(\lambda\wedge f^{-1}(\underline{B}_\lambda)(x)),\bigvee_{\lambda\in[-1,0]}(\lambda\wedge f^{-1}(\overline{B}_\lambda)(x)))$$
$$=(\bigvee_{\lambda\in[-1,0]}(\lambda\wedge\underline{B}_\lambda(f(x)),\bigvee_{\lambda\in[-1,0]}(\lambda\wedge\overline{B}_\lambda(f(x)))$$
$$=(\underline{B}(f(x)),\overline{B}(f(x)))$$
$$=f^{-1}(\underline{B},\overline{B})(x)$$

b)设 $\lambda\in[0,1]$ $\forall x\in X^0\cup X^+$,可类似证明。

定理2设映射 $f:X\to Y$, $x\mapsto y$

1)若 $(\underline{A},\overline{A})\in RBBFS(X)$,则 $f(\underline{A},\overline{A})=(\bigcup_{\lambda\in[-1,1]}\lambda f(\underline{A}_{\dot\lambda}),\bigcup_{\lambda\in[-1,1]}\lambda f(\overline{A}_{\dot\lambda}))$

2)若 $(\underline{B},\overline{B})\in RBBFS(Y)$,则 $f^{-1}(\underline{B},\overline{B})=(\bigcup_{\lambda\in[-1,1]}\lambda f^{-1}(\underline{B}_{\dot\lambda}),\bigcup_{\lambda\in[-1,1]}\lambda f^{-1}(\overline{B}_{\dot\lambda}))$

定理3设映射 $f:X\to Y$, $x\mapsto y$

1)若 $(\underline{A},\overline{A})\in RBBFS(X)$,且 $\underline{A}_{\dot\lambda}\subseteq\underline{H}_A(\lambda)\subseteq\underline{A}_\lambda$, $\overline{A}_{\dot\lambda}\subseteq\overline{H}_A(\lambda)\subseteq\overline{A}_\lambda$,则

$$f(\underline{A},\overline{A})=(\bigcup_{\lambda\in[-1,1]}\lambda f(\underline{H}_A(\lambda)),\bigcup_{\lambda\in[-1,1]}\lambda f(\overline{H}_A(\lambda)))$$

2)若 $(\underline{B},\overline{B})\in RBBFS(Y)$,且 $\underline{B}_{\dot\lambda}\subseteq\underline{H}_B(\lambda)\subseteq\underline{B}_\lambda$, $\overline{B}_{\dot\lambda}\subseteq\overline{H}_B(\lambda)\subseteq\overline{B}_\lambda$ 则

$$f^{-1}(\underline{B},\overline{B})=(\bigcup_{\lambda\in[-1,1]}\lambda f^{-1}(\underline{H}_B(\lambda)),\bigcup_{\lambda\in[-1,1]}\lambda f^{-1}(\overline{H}_B(\lambda)))$$

由定理1,定理2可直接得出定理3。

定理4设映射 $f:X\to Y$, $x\mapsto y$

1)若 $(\underline{A},\overline{A})\in RBBFS(X)$,则 $f(\underline{A},\overline{A})=(\bigcap_{\lambda\in[-1,1]}\lambda\cdot f(\underline{A}_\lambda),\bigcap_{\lambda\in[-1,1]}\lambda\cdot f(\overline{A}_\lambda))$

2)若 $(\underline{B},\overline{B})\in RBBFS(Y)$,则 $f^{-1}(\underline{B},\overline{B})=(\bigcap_{\lambda\in[-1,1]}\lambda\cdot f^{-1}(\underline{B}_\lambda),\bigcap_{\lambda\in[-1,1]}\lambda\cdot f^{-1}(\overline{B}_\lambda))$

证明1)a)设 $\lambda\in[-1,0]$, $\forall y\in Y^-\cup Y^0$

$$f(\underline{A},\overline{A})(y)=(\bigcap_{\lambda\in[-1,0]}\lambda\cdot f(\underline{A}_\lambda),\bigcap_{\lambda\in[-1,0]}\lambda\cdot f(\overline{A}_\lambda))(y)$$
$$=(\bigcap_{\lambda\in[-1,0]}\lambda\cdot f(\underline{A}_\lambda)(y),\bigcap_{\lambda\in[-1,0]}\lambda\cdot f(\overline{A}_\lambda)(y))$$
$$=(\bigwedge_{\lambda\in[-1,0]}(\lambda\vee f(\underline{A}_\lambda)(y)),\bigwedge_{\lambda\in[-1,0]}(\lambda\vee f(\overline{A}_\lambda)(y)))$$
$$=(\bigwedge_{\lambda\in[-1,0]}(\lambda\vee(\bigvee_{f(x)=y}\underline{A}_\lambda(x))),\bigwedge_{\lambda\in[-1,0]}(\lambda\vee(\bigvee_{f(x)=y}\overline{A}_\lambda(x))))$$

下证 $\bigwedge_{\lambda\in[-1,0]}(\lambda\vee(\bigvee_{f(x)=y}\underline{A}_\lambda(x)))=\bigvee_{f(x)=y}(\bigwedge_{\lambda\in[-1,0]}(\lambda\vee\underline{A}_\lambda(x)))$

显然,$\lambda\vee(\bigvee_{f(x)=y}\underline{A}_\lambda(x))\ge\lambda\vee\underline{A}_\lambda(x)$

所以,$\bigwedge_{\lambda\in[-1,0]}(\lambda\vee(\bigvee_{f(x)=y}\underline{A}_\lambda(x)))\ge\bigwedge_{\lambda\in[-1,0]}(\lambda\vee\underline{A}_\lambda(x))$

从而,$\bigwedge_{\lambda\in[-1,0]}(\lambda\vee(\bigvee_{f(x)=y}\underline{A}_\lambda(x)))\ge\bigvee_{f(x)=y}(\bigwedge_{\lambda\in[-1,0]}(\lambda\vee\underline{A}_\lambda(x)))$

下证“=”成立,若不然,存在 $\alpha\in[-1,0]$,有

$$\bigwedge_{\lambda\in[-1,0]}(\lambda\vee(\bigvee_{f(x)=y}\underline{A}_\lambda(x)))>\alpha>\bigvee_{f(x)=y}(\bigwedge_{\lambda\in[-1,0]}(\lambda\vee\underline{A}_\lambda(x)))$$

而 $\bigvee_{f(x)=y} \underline{A}_\alpha(x) \in \{-1,0\}$ 。

i)若 $\bigvee_{f(x)=y} \underline{A}_\alpha(x) = -1$

$\alpha < \bigwedge_{\lambda\in[-1,0]}(\lambda \vee (\bigvee_{f(x)=y} \underline{A}_\lambda(x))) < \alpha \vee (\bigvee_{f(x)=y} \underline{A}_\alpha(x)) = \alpha$

矛盾。

ii)若 $\bigvee_{f(x)=y} \underline{A}_\alpha(x) = 0$,则存在 x_0,有 $f(x_0)=y$ 且 $\underline{A}_\alpha(x_0) = 0$,故对任意 $\lambda \geqslant \alpha$,

有 $\underline{A}_\lambda(x_0) = 0$,

$\alpha > \bigvee_{f(x)=y}(\bigwedge_{\lambda\in[-1,0]}(\lambda \vee \underline{A}_\lambda(x_0)))$

$\geq \bigwedge_{\lambda\in[-1,0]}(\lambda \vee \underline{A}_\lambda(x_0))$

$= (\bigwedge_{\lambda\in[-1,\alpha]}(\lambda \vee \underline{A}_\lambda(x_0))) \wedge (\bigwedge_{\lambda\in[\alpha,0]}(\lambda \vee \underline{A}_\lambda(x_0)))$

$= (\bigwedge_{\lambda\in[-1,\alpha]}(\lambda \vee \underline{A}_\lambda(x_0))) \geq \bigwedge_{\lambda\in[-1,\alpha]} \lambda = \alpha$

矛盾。

从而 $\bigwedge_{\lambda\in[-1,0]}(\lambda \vee (\bigvee_{f(x)=y} \underline{A}_\lambda(x))) = \bigvee_{f(x)=y}(\bigwedge_{\lambda\in[-1,0]}(\lambda \vee \underline{A}_\lambda(x)))$

类似可证 $\bigwedge_{\lambda\in[-1,0]}(\lambda \vee (\bigvee_{f(x)=y} \overline{A}_\lambda(x))) = \bigvee_{f(x)=y}(\bigwedge_{\lambda\in[-1,0]}(\lambda \vee \overline{A}_\lambda(x)))$

故 $\lambda \in[-1,0]$时,

$(\bigcap_{\lambda\in[-1,0]} \lambda \cdot f(\underline{A}_\lambda), \bigcap_{\lambda\in[-1,0]} \lambda \cdot f(\overline{A}_\lambda))(y) = (\bigvee_{f(x)=y} \underline{A}(x), \bigvee_{f(x)=y} \overline{A}(x)) = f(\underline{A}, \overline{A})(y)$

b)设 $\lambda \in[0,1]$, $\forall y \in Y^0 \cup Y^+$可类似证明。

2)a)设 $\lambda \in[-1,0]$, $\forall x \in X^- \cup X^0$,

$= ((\bigcap_{\lambda\in[-1,0]} \lambda \cdot f^{-1}(\underline{B}_\lambda))(x), (\bigcap_{\lambda\in[-1,0]} \lambda \cdot f^{-1}(\overline{B}_\lambda))(x))$

$= ((\bigwedge_{\lambda\in[-1,0]} \lambda \cdot f^{-1}(\underline{B}_\lambda))(x), (\bigwedge_{\lambda\in[-1,0]} \lambda \cdot f^{-1}(\overline{B}_\lambda))(x))$

$= (\bigwedge_{\lambda\in[-1,0]}(\lambda \vee f^{-1}(\underline{B}_\lambda)(x)), \bigwedge_{\lambda\in[-1,0]}(\lambda \vee f^{-1}(\overline{B}_\lambda)(x)))$

$= (\bigwedge_{\lambda\in[-1,0]}(\lambda \vee \underline{B}_\lambda(f(x)), \bigwedge_{\lambda\in[-1,0]}(\lambda \vee \overline{B}_\lambda(f(x)))$

$= (\underline{B}(f(x)), \overline{B}(f(x)))$

$= f^{-1}(\underline{B}, \overline{B})(x)$

b)设 $\lambda \in[0,1]$, $\forall x \in X^0 \cup X^+$,可类似证明。

定理5设映射 $f: X\to Y$, $x \mapsto y$

1)若 $(\underline{A}, \overline{A}) \in \mathrm{RBBFS}(X)$,则 $f(\underline{A}, \overline{A}) = (\bigcap_{\lambda\in[-1,1]} \lambda \cdot f(\underline{A}_{\dot\lambda}), \bigcap_{\lambda\in[-1,1]} \lambda \cdot f(\overline{A}_{\dot\lambda}))$

2)若 $(\underline{B}, \overline{B}) \in \mathrm{RBBFS}(Y)$,则 $f^{-1}(\underline{B}, \overline{B}) = (\bigcap_{\lambda\in[-1,1]} \lambda \cdot f^{-1}(\underline{B}_{\dot\lambda}), \bigcap_{\lambda\in[-1,1]} \lambda \cdot f^{-1}(\overline{B}_{\dot\lambda}))$

定理6设映射 $f: X\to Y$, $x \mapsto y$

1)若 $(\underline{A}, \overline{A}) \in \mathrm{RBBFS}(X)$,且 $\underline{A}_{\dot\lambda} \subseteq \underline{H}_A(\lambda) \subseteq \underline{A}_\lambda$, $\overline{A}_{\dot\lambda} \subseteq \overline{H}_A(\lambda) \subseteq \overline{A}_\lambda$,则

$f(\underline{A}, \overline{A}) = (\bigcap_{\lambda\in[-1,1]} \lambda \cdot f(\underline{H}_A(\lambda)), \bigcap_{\lambda\in[-1,1]} \lambda \cdot f(\overline{H}_A(\lambda)))$

2)若 $(\underline{B}, \overline{B}) \in \mathrm{RBBFS}(Y)$,且 $\underline{B}_{\dot\lambda} \subseteq \underline{H}_B(\lambda) \subseteq \underline{B}_\lambda$, $\overline{B}_{\dot\lambda} \subseteq \overline{H}_B(\lambda) \subseteq \overline{B}_\lambda$,则

$f^{-1}(\underline{B}, \overline{B}) = (\bigcap_{\lambda\in[-1,1]} \lambda \cdot f^{-1}(\underline{H}_B(\lambda)), \bigcap_{\lambda\in[-1,1]} \lambda \cdot f^{-1}(\overline{H}_B(\lambda)))$

由定理4,定理5可直接得出定理6。

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Extension Theorem of Rough Both-branch Fuzzy Sets

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Abstract: This paper presents extension principle of rough Both-branch fuzzy sets by using representation theorem of rough Both-branch fuzzy sets and provides a way for the knowledge representation.

Key words: Rough Both-branch fuzzy sets; Representation theorem; Extension principle

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Relations and Pollution Assessment of the Cd, Cu Content of Soil and Rapeseed in Liangshan

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Abstract: The Cd, Cu content of soil and the corresponding rapeseed in Liangshan was tested in the experiment. Results show as follows: Assessed by nemerow pollution index. The Cd content in the rapseed producing area has achieved light pollution degree and the Cu content in there is of the clean(safe)degree. Tht Cd content in rapeseed is at the light pollution degree while the Cu content of that is at clean(safe)degree. There is no significant positive correlation among the soil, rapeseed Cd and Cu.

Key words: Cadmium; Copper; Pollution; Nemerow pollution index