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Banach空间中有限簇拟压缩映射的迭代逼近*

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【摘 要】在一实的Banach空间中,引入一修订的有限簇拟压缩映像$T_1,T_2,\cdots,T_m$,并证明了在一定条件下,关于$\{x_n\}$的迭代:$x_{n+1}=(1-\alpha_{1n})x_n+\alpha_{1n}T_1y_{1n}+u_{1n}$,$y_{1n}=(1-\alpha_{2n})x_n+\alpha_{2n}T_2y_{2n}+u_{2n}\cdots$,$y_{(m-1)n}=(1-\alpha_{mn})x_n+\alpha_{mn}T_mx_n+u_{mn}$,$(m\geqslant 2)$强收敛与有限个似压缩簇$T_1,T_2,\cdots,T_m$的公共不动点。本文的结果改进和推广了一些文献的最新结果。

【关键词】拟压缩映象;一致光滑Banach空间;具误差的Ishikawa迭代;正规对偶映象;不动点
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1 引言及预备知识

设E是实Banach空间,E^*是E的对偶空间.$\langle\cdot,\cdot\rangle$表示E和$E^*$的广义对偶对.$J:E\to E^*$是由下列式子定义的正规对偶映射:

$J(x)=\{f\in E^*:\langle x,f\rangle=\|x\|\cdot\|f\|,\|f\|=\|x\|,x\in X\}$

首先给出一些基本概念.

定义1 映射$T:E\to E$称为拟压缩映射,如果对所有的$x,y\in E$,有

$\|Tx-Ty\|\leqslant k\max\{\|x-y\|,\|x-Ty\|,\|y-Tx\|,\|x-Tx\|,\|y-Ty\|\}$,$0<k<1$成立.

定义2 增生映象

设$E\to E^*$是正规对偶映象,$T:D(T)\subset E\to 2^E$是一多值映象.

(a)T称为增生的,如果对任意$x,y\in D(T)$,存在$j(x-y)\in J(x-y)$,使得

$\langle u-\nu,j(x-y)\rangle\geqslant 0,\forall u\in T(x),\nu\in T(y)$.

(b)T称为伪压缩的,如果对任意$x,y\in D(T)$,存在$j(x-y)\in J(x-y)$,使得

$\langle u-\nu,j(x-y)\rangle\leqslant\|x-y\|^2,\forall u\in T(x),\nu\in T(y)$.

注.易知映象T是增生的,当且仅当I−T是伪压缩的,其中I是恒等映射.下面给出迭代序列.

(1)设D是E的一个非空子集.$T:D\to 2^E$是一映射.$x_0\in D$是一给定的点.如果下式定义的序列$\{x_n\}\subset D$,

$x_{n+1}=(1-\alpha_n)x_n+\alpha_nTy_n$,

$y_n=(1-\beta_n)x_n+\beta_nTx_n,(n\geqslant 0)$.

则称$\{x_n\}$为T的Ishikawa迭代序列[1],其中$\{\alpha_n\}$,$\{\beta_n\}$是[0,1]间满足某些条件的实数列.

下面,文献[3]给出了一种具有误差的迭代序列如下:

(2)设D是E的一个非空子集.$T:D\subset E\to E$,对$\forall x_0\in D$,$\{x_n\}$由下列迭代序列定义:

$x_{n+1}=(1-\alpha_n)x_n+\alpha_nTy_n+u_n$,

$y_n=(1-\beta_n)x_n+\beta_nTy_n+\nu_n,n\geqslant 0$.

其中$\{\alpha_n\}$,$\{\beta_n\}$是[0,1]间的实数列.$\{u_n\}$,$\{\nu_n\}$是E中的序列,称$\{x_n\}$为T的具有误差的Ishikawa迭代序列.

本文的目的是在一般的实的Banach空间中,引入了一修订的有限簇拟压缩映像$T_1,T_2,\cdots,T_m$.并证明了有限簇拟压缩映像在Banach空间的强收敛定理.即证明了迭代序列$\{x_n\}$强收敛于$T_1,T_2,\cdots,T_m$的公共不动点.

为了证明本文的主要结果,我们需要下面的引理:

引理1[4] 设E是实Banach空间,J是$E\to E^*$的正规对偶映像.则对$\forall x,y\in E$.有

$\|x+y\|^2\leqslant\|x\|^2+2\langle y,j(x+y)\rangle,\forall j(x+y)\in J(x+y)$.

引理2[5] 设E是一致光滑的Banach空间当且仅当J是单值的且在E的任一有界子集上连续.

引理3[6] 令a_n,b_n,c_n是三个非负实数列.满足

$a_{n+1}\leqslant(1-t_n)a_n+b_n+c_n,n\geqslant 0$

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其中$t_n\in[0,1]$，$\sum t_n=+\infty$，$b_n=O(t_n)$，$\sum c_n<+\infty$. 则
$\lim\limits_{n\to\infty} a_n=0$.

2 主要结果

定理1 设E是任意Banach空间，$T_i:E\to E(i=1,2,\cdots m)$是m个一致连续的拟压缩映象，$-T_i$是增生的，$F(T_i)$表示$T_i$的不动点集，且$\bigcap\limits_{i=1}^{m}F(T_i)$不为空集，$x^*$是$\bigcap\limits_{i=1}^{m}F(T_i)$的不动点. $\{\alpha_{in}\}\subset[0,1]$. $i=1,2,\cdots,m$. $\{u_n\}$，$\{\nu_n\}$是E中的序列且满足：

$\lim\limits_{n\to\infty} a_{in}=0, i=1,2,\cdots m$；$\sum\limits_{n=0}^{\infty}\alpha_{in}=+\infty, i=1,2,\cdots m$；$\sum\limits_{n=0}^{\infty}\|u_{in}\|<\infty, i=1,2,\cdots m-1$；$\lim\limits_{n\to\infty}\|u_{mn}\|=0$.

任取$x_0\in D$，定义$\{x_n\}$的迭代序列为:

$x_{n+1}=(1-\alpha_{1n})x_n+\alpha_{1n}T_1y_{1n}+u_{1n}$,

$y_{1n}=(1-\alpha_{2n})x_n+\alpha_{2n}T_2y_{2n}+u_{2n}$, (*)

………………

$y_{(m-1)n}=(1-\alpha_{mn})x_n+\alpha_{mn}T_mx_n+u_{mn}$, $(m\geqslant 2)$

如果$\{T_iy_{in}\}$，$i=1,2,\cdots m$有界，则$\{x_n\}$强收敛到x^*.

证明 先证$\{x_n\}$的有界性.

事实上，由T_i的连续性和$-T_i$是增生的. 知T_i有唯一不动点，设为x^*. 因为$-T_i$是增生的. 对$\forall x,y\in E$，$\exists j(x+y)\in J(x+y)$. 使得

$\langle T_ix-T_iy, j(x+y)\rangle=-\langle(-T_i)x-(-T_i)y, j(x+y)\rangle\leqslant 0$.

上式等价于$\forall\lambda>0$，及$\forall x,y\in E$，

$\|x-y\|\leqslant\|x-y-\lambda(T_i-T_iy)\|$.

由(*)式，

$x_n=x_{n+1}+\alpha_{1n}x_n-\alpha_{1n}T_1y_{1n}-u_{1n}$

$=(1+\alpha_{1n})x_{n+1}-\alpha_{1n}T_1x_{n+1}+\alpha_{1n}^2(x_n-T_1y_{1n})+\alpha_{1n}(T_1x_{n+1}-T_1y_{1n}-u_{1n})-u_{1n}$.

$x^*=(1+\alpha_{1n})x^*-\alpha_{1n}T_1x^*$.

$x_n-x^*=(1+\alpha_{1n})(x_{n+1}-x^*)-\alpha_{1n}(T_1x_{n+1}-T_1x^*)+\alpha_{1n}^2(x_n-T_1y_{1n})+\alpha_{1n}(T_1x_{n+1}-T_1y_{1n}-u_{1n})$

$$\|x_{n+1}-x^*\|\leqslant\frac{1}{1+\alpha_{1n}}[\|x_n-x^*\|+\alpha_{1n}^2\|x_n-T_1y_{1n}\|+\alpha_{1n}\|T_1x_{n+1}-T_1y_{1n}-u_{1n}\|+\|u_{1n}\|]$$

$$\leqslant\frac{1}{1+\alpha_{1n}}\|x_n-x^*\|+\alpha_{1n}^2\|x_n-T_1y_{1n}\|+\alpha_{1n}\|T_1x_{n+1}-T_1y_{1n}-u_{1n}\|+\|u_{1n}\|.$$

由$\{T_iy_{in}\}$，$i=1,2,\cdots m$的有界性，令

$d=\max\{\sup\{\|T_iy_{in}-x^*\|\},\sup\{\|T_2y_{2n}-x^*\|\},\cdots,\sup\{\|T_my_{mn}-x^*\|\},n\geqslant 0\}$.

$M=d+\sum\limits_{n=0}^{\infty}\|u_{1n}\|+1$.

对于$\|x_n-x^*\|$，由(*)有

$x_1=(1-\alpha_{10})x_0+\alpha_{10}T_1y_{10}+u_{10}$,

$\|x_1-x^*\|\leqslant(1+\alpha_{10})\|x_0-x^*\|+\alpha_{10}\|T_1y_{10}-x^*\|+\|u_{10}\|\leqslant d+\|u_{10}\|$.

则对$\forall n\geqslant 0$，有

$$\|x_n-x^*\|\leqslant d+\sum_{i=0}^{n-1}\|u_{1i}\|\leqslant M \tag{3}$$

又T_i是拟压缩映射，则对$\forall x\in D(T_i)$，有

$$\|T_ix-x^*\|=\|T_ix-T_ix^*\|\leqslant\frac{k}{1-k}\|x-x^*\|, 0<k<1. \tag{4}$$

$$由\|x_n-T_1y_{ln}\|\leqslant\|x_n-y_{ln}\|+\|y_{ln}-T_1y_{ln}\|. \tag{5}$$

因为$\|x_n-y_{ln}\|\leqslant\alpha_{2n}\|x_n-T_2y_{2n}\|+\|u_{2n}\|\leqslant\alpha_{2n}[\alpha_{3n}\|x_n-T_3y_{3n}\|+\|u_{3n}\|]+\|u_{2n}\|$

$=\alpha_{2n}\alpha_{3n}\|x_n-T_3y_{3n}\|+\alpha_{2n}\|u_{3n}\|+\|u_{2n}\|$

$\leqslant\cdots\leqslant\alpha_{2n}\alpha_{3n}\cdots\alpha_{mn}\|x_n-T_mx_n\|+\alpha_{2n}\cdots\alpha_{(m-1)n}\|u_{mn}\|+\cdots+\alpha_{2n}\|u_{3n}\|+\|u_{2n}\|$.

不妨设$\alpha_n=\max\{\alpha_{1n},\alpha_{2n},\cdots,\alpha_{mn}\}$，$u_n=\max\{u_{1n},u_{2n},\cdots,u_{mn}\}$.

则$\|x_n-y_{ln}\|\leqslant\alpha_{2n}\alpha_{3n}\cdots\alpha_{mn}\|x_n-T_mx_n\|+\alpha_{2n}\cdots\alpha_{(m-1)n}\|u_{mn}\|+\cdots+\alpha_{2n}\|u_{3n}\|+\|u_{2n}\|$

$$\leqslant\alpha_n^{m-1}\|x_n-T_mx_n\|+\frac{\|u_n\|(1-\alpha_n^{m-1})}{1-\alpha_n}$$

$$\leq \alpha_n^{m-1}[\|x_n-x^*\|+\|T_mx_n-x^*\|]+\frac{\|u_n\|(1-\alpha_n^{m-1})}{1-\alpha_n}$$

由(3),(4)式易知 $\alpha_n^{m-1}[\|x_n-x^*\|+\|T_mx_n-x^*\|]+\frac{\|u_n\|(1-\alpha_n^{m-1})}{1-\alpha_n}$

$$\leq \alpha_n^{m-1}[M+\frac{k}{1-k}\|x_n-x^*\|+\frac{\|u_n\|(1-\alpha_n^{m-1})}{1-\alpha_n}$$

$$\leq \alpha_n^{m-1}\frac{k}{1-k}M+\|x_n-x^*\|+\frac{\|u_n\|(1-\alpha_n^{m-1})}{1-\alpha_n} \tag{7}$$

又 $\|y_{1n}-T_1y_{1n}\|\leq\|T_1y_{1n}-x^*\|+\|y_{1n}-x^*\|\leq M+\|y_{1n}-x^*\|$ (8)

因为 $\|y_{1n}-x^*\|=\|(1-\alpha_{2n})(x_n-x^*)+\alpha_{2n}(T_2y_{2n}-x^*)+u_{2n}\|$

$$\leq(1-\alpha_{2n})\|x_n-x^*\|+\alpha_{2n}\|T_2y_{2n}-x^*\|+\|u_{2n}\|$$

$$\leq(1-\alpha_{2n})M+\alpha_{2n}[(1-\alpha_{3n})M+\alpha_{3n}\|T_3y_{3n}-x^*\|+\|u_{3n}\|]+\|u_{2n}\|$$

$$=(1-\alpha_{2n})M+\alpha_{2n}(1-\alpha_{3n})M+\alpha_{2n}\alpha_{3n}\|T_3y_{3n}-x^*\|+\alpha_{2n}\|u_{3n}\|]+\|u_{2n}\|$$

$$\leq\cdots\leq(1-\alpha_{2n})M+\cdots+\alpha_{2n}\alpha_{3n}\cdots\alpha_{(m-1)n}M+\alpha_{2n}\alpha_{3n}\cdots\alpha_{mn}\|T_mx_n-x^*\|$$

$$+\alpha_{2n}\alpha_{3n}\cdots\alpha_{(m-1)n}\|u_{mn}\|+\cdots+\|u_{2n}\|$$

$$\leq\frac{(1-\alpha_n)M(1-\alpha_n^{m-1})}{1-\alpha_n}+\alpha_n^{m-1}\frac{1}{1-k}M+\frac{\|u_n\|(1-\alpha_n^{m-1})}{1-\alpha_n} \tag{9}$$

对于 $\|x_{n+1}-y_{1n}\|\leq(1-\alpha_{1n})\|x_n-y_{1n}\|+\alpha_{1n}\|T_1y_{1n}-y_{1n}\|+\|u_{1n}\|$,

由(7),(8),(9)知

$(1-\alpha_{1n})\|x_n-y_{1n}\|+\alpha_{1n}\|T_1y_{1n}-y_{1n}\|+\|u_{1n}\|$

$$\leq(1-\alpha_{1n})\left[\frac{1}{1-k}\alpha_n^{m-1}M+\frac{\|u_n\|(1-\alpha_n^{m-1})}{1-\alpha_n}\right]+\alpha_n\left[M+M(1-\alpha_n^{m-1})+\frac{1}{1-k}\alpha_n^{m-1}M+\frac{\|u_n\|(1-\alpha_n^{m-1})}{1-\alpha_n}\right]$$

由假设条件知上式当 $n\to+\infty$ 时,

$$(1-\alpha_{1n})\left[\frac{1}{1-k}\alpha_n^{m-1}M+\frac{\|u_n\|(1-\alpha_n^{m-1})}{1-\alpha_n}\right]+\alpha_n\left[M+M(1-\alpha_n^{m-1})+\frac{1}{1-k}\alpha_n^{m-1}M+\frac{\|u_n\|(1-\alpha_n^{m-1})}{1-\alpha_n}\right]\to 0$$

故 $\|x_{n+1}-y_{1n}\|\to0$,根据 T_i 的一致连续性,我们有

$\|T_1x_{n+1}-T_1y_{1n}\|\to0,(n\to\infty)$

故

$$\|x_{n+1}-x^*\|\leq(1-\frac{\alpha_n}{1+\alpha_n})\|x_n-x^*\|+\alpha_n^2\left[2M\frac{1}{1-k}\alpha_n^{m-1}+2\frac{\|u_n\|(1-\alpha_n^{m-1})}{1-\alpha_n}+M(1-\alpha_n^{m-1})\right]$$

$$+\alpha_n\|T_1x_{n+1}-T_1y_{1n}\|+(1-\alpha_n)\|u_n\|$$

$$\leq(1-\alpha_n)\|x_n-x^*\|+2M\frac{1}{1-k}\alpha_n^{m+1}+2\alpha_n^2\frac{\|u_n\|(1-\alpha_n^{m-1})}{1-\alpha_n}+M(1-\alpha_n^{m-1})\alpha_n^2$$

$$+\alpha_n\|T_1x_{n+1}-T_1y_{1n}\|+(1-\alpha_n)\|u_n\|.$$

上式中令 $\alpha_n=\|x_n-x^*\|$, $t_n=\alpha_n$,

$$b_n=2M\frac{1}{1-k}\alpha_n^{m+1}+2\alpha_n^2\frac{\|u_n\|(1-\alpha_n^{m-1})}{1-\alpha_n}+M(1-\alpha_n^{m-1})\alpha_n^2+\alpha_n\|T_1x_{n+1}-T_1y_{1n}\|$$

$c_n=(1-\alpha_n)\|u_n\|$.

则由引理3,知当 $n\to\infty$ 时, $\|x_n-x^*\|\to0$. 证毕.

注　定理1将文献[8]的结果推广到了有限个且在一定的条件下,结果仍成立.

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（ii）∀a∈Z且(p,a)从而a≡0(modp)不成立，即[a]≠[0]，从而[a]∈Z_p^+；

（iii）由推论6.1知：$[a]^{p-1}=[1]$，从而$[a^{p-1}]=[1]$，因此$a^{p-1}\equiv 1(modp)$。

最后指出，欧拉——费马定理常表现为：(a是任意整数，p是素数)$a^p\equiv a(modp)$。

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One Application about Rough Algebra

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Abstract: In this paper，we introduced some basic conceptions of semi-group and the important theorem5.1 firstly. And then，we proved that $Z_m=\{[0],[1],\cdots[m-1]\}$ is a monoid semi-group in the binary Operation [a][b]=[ab]，consequently，Z_m^+ is a group of (m-1) order. Finally，we obtained the inference 6.1，and finished this proof.

Key words: Euler-Fermat；Equivalence relation；M-congruence modulo

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The Iterative Approximation of Limited Cluster Quasi-Contractive Mapping in Banach Space

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Abstract: In a solid Banach space，the introduction of a revised limited cluster $T_1,T_2,\cdots,T_m$ quasi-contractive mapping was done，and proved that under certain conditions，on the $\{x_n\}$ of iterations：$x_{n+1}=(1-\alpha_{1n})x_n+\alpha_{1n}T_1y_{1n}+u_{1n}$，$y_{1n}=(1-\alpha_{2n})x_n+\alpha_{2n}T_2y_{2n}+u_{2n}\cdots$，$y_{(m-1)n}=(1-\alpha_{mn})x_n+\alpha_{mn}T_mx_n+u_{mn}$，$(m\geqslant 2)$ strong convergence in a finite number of clusters to be compression of the common fixed point of $T_1,T_2,\cdots,T_m$. The results of this paper improve and generalize of the latest results of the literature.

Key words: Quasi-contractive mapping；Consistent smooth Banach space；An error of Ishikawa iterations；Normalized duality mapping；Fixed point