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非线性混合拟似变分不等式的逼近迭代原理

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【摘 要】利用辅助原理技术，建立和分析了求解非线性混合拟似变分不等式的逼近迭代原理，原理的收敛性仅需映象的伪单调性，此性质比单调性更弱。这种原理的收敛性结果推广了文献中某些已有结果。

【关键词】辅助原理技术；伪单调性；逼近迭代原理；反对称性

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Proximal Iterative Methods for Solving Nonlinear Mixed Quasivariational-Like Inequalities

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Abstract: In this paper, by applying the auxiliary principle technique, a proximal iterative method for solving nonlinear mixed quasivariational-like inequalities is suggested and analyzed. The convergence of the method requires only the pseudo-monotonicity of mappings, which is weaker condition than the monotonicity. This method and convergence results generalize some results in literatures.

Key words: Auxiliary principle technique; Pseudo- monotonicity; Proximal iterative method; Skew-symmetric functions.

1 Preliminaries

Let H be a real Hilbert space with norm and inner product denoted by $\|\cdot\|$ and $<\cdot,\cdot>$, respectively, I stand for the identity mapping on H. Let $CB(H)$ be the families of all the nonempty bounded closed subsets of H. Let $T, A: H\to H$ and $N, \eta: H\times H\to H$ be single-valued mappings and $\varphi: H\times H\to R\cup\{+\infty\}$ be a real function. We consider the following nonlinear mixed quasivariational-like ine-qualities problem: find $u\in H$ such that

$$<N(Tu,Au),\eta(g(\nu),g(u))>+\varphi(g(\nu),g(u))-\varphi(g(u),g(u))\geq 0,\ \forall g(\nu)\in H \tag{1.1}$$

(1) If $g\equiv I$, $\varphi: H\to R\cup\{+\infty\}$, then the problem (1.1) reduces to the following nonlinear mixed variational-like inequalities problem: find $u\in H$ such that

$$<N(Tu,Au),\eta(\nu,u)>+\varphi(\nu)-\varphi(u)\geq 0,\ \forall \nu\in H \tag{1.2}$$

The problem (1.2) was introduced and studied by Ding [1].

(2) If $g\equiv I$, $N(Tu,Au)=Tu$, $\eta(\nu,u)=\nu-u$, then the problem (1.1) reduces to the following nonlinear mixed quasivariational-like inequalities problem: find $u\in H$ such that

$$<Tu,\nu-u>+\varphi(\nu,u)-\varphi(u,u)\geq 0,\quad \forall \nu\in H \tag{1.3}$$

The problem (1.3) was introduced and studied by M.A.Noor[2],[3].

(3) If $N(Tu,Au)=Tu$, $\eta(\nu,u)=g(\nu)-g(u)$, $\forall u,\nu\in H$, $\varphi: H\times H\to R\cup\{+\infty\}$ then the problem (1.1) re-

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duces to the following nonlinear mixed variational inequalities problem: find $u \in H$ such that

$$<Tu, g(v)-g(u)> + \varphi(g(v)) - \varphi(g(u)) \geq 0, \quad \forall g(v) \in H \tag{1.4}$$

The problem (1.4) was introduced and studied by Noor[4].

It is easy to see that the problem (1.1) includes a number of extensions and generalizations of many variational and variational-like inequalities in literature as special cases.

Lemma 1.1[3] For all $u, v \in H$, we have

$$2<u, v> = \| u+v \|^2 - \| u \|^2 - \| v \|^2$$

Definition 1.1[3] The bifunction $\varphi(\cdot,\cdot): H \times H \to R \cup \{+\infty\}$ is called skew-symmetric, if it satisfies

$$\varphi(u, u) - \varphi(u, v) - \varphi(v, u) + -\varphi(v, v) \geq 0, \quad \forall u, v \in H$$

It is easy to see that, if the bifunction $\varphi(\cdot,\cdot)$ is linear in both the variables, then $\varphi(\cdot,\cdot)$ is non-negative.

Definition 1.2 Let $T, g: H \to H$ and $N, \eta: H \times H \to H$ be single-valued mappings.

i) $T: H \to H$ is said to be η-g-monotone with respect to the first argument of $N(\cdot,\cdot)$, if

$$<N(Tu, \cdot) - N(Tv, \cdot), \eta(g(u), g(v))> \geq 0, \quad \forall u, v \in H$$

ii) $T: H \to H$ is said to be η-g-pseudomonotone with respect to the first argument of $N(\cdot,\cdot)$, if

$$<N(Tu, \cdot), \eta(g(v), g(u))> \geq 0 \Rightarrow <N(Tv, \cdot), \eta(g(v), g(u))> \geq 0, \quad \forall u, v \in H$$

Similarly, we can define the η-g-pseudomonotone of $T: H \to H$ with respect to the second argument of $N(\cdot,\cdot)$. It is known that monotonicity implies pseudomonotonicity, but the converse is not true, see[5].

We observe that if $T: H \to H$ and $A: H \to H$ are η-g-pseudomonotone with respect to the first and second argument of $N(\cdot,\cdot)$, respectively, then

$$<N(Tu, Au), \eta(g(v), g(u))> \geq 0 \Rightarrow N(Tv, Av), \eta(g(v), g(u))> \geq 0, \quad \forall u, v \in H$$

2 Iterative algorithm of solutions

In this section, we suggest and analyze a new iterative algorithm for solving the problem (1.1) by using the auxiliary variational inequalities technique. For a given $u \in H$, consider the problem of finding a unique $w \in H$ satisfying the following auxiliary variational inequality:

$$<g(w)-g(u), g(v)-g(w)> + <\rho N(Tw, Aw), \eta(g(v), g(w))> + \rho\varphi(g(v), g(w)) - \rho\varphi(g(w), g(w)) \geq 0 \quad \forall g(v) \in H \tag{2.1}$$

where $\rho > 0$ is a constant.

We note that if $w=u$, then clearly w is a solution of the problem (1.1). By observation, we suggest the following iterative algorithm for solving the problem (1.1).

Algorithm 2.1 For a given $u_0 \in H$, compute the approximate solution u_{n+1} by the iterative scheme

$$<g(u_{n+1})-g(u_n), g(v)-g(u_{n+1})> + \rho N(Tu_{n+1}, Au_{n+1}), \eta(g(v), g(u_{n+1}))> + \rho\varphi(g(v), g(u_{n+1})) - \rho\varphi(g(u_{n+1}), g(u_{n+1})) \geq 0, \quad \forall v \in H \tag{2.2}$$

Note that if $g \equiv I, \varphi: H \to R \cup \{+\infty\}$, then Algorithm 2.1 reduces to, which is a new iterative algorithm for the problem (1.2).

3 Existence and convergence result

Lemma 3.1 Let $\bar{u} \in H$ be the exact solution of the problem (1.1) and u_{n+1} be the approximate solution obtained from Algorithm 2.1. If operator $T: H \to H$ and $A: H \to H$ are η-g-pseudomonotone with respect to the first and second argument of $N(\cdot,\cdot)$, respectively. Suppose $\eta(u, v) = -\eta(v, u)\ \forall u, v \in H$, and $\varphi(.,.): H \times H \to R \cup \{+\infty\}$ is skew-symmetric, then

$$\| g(u_{n+1}) - g(\bar{u}) \|^2 \leq \| g(u_n) - g(\bar{u}) \|^2 - \| g(u_{n+1}) - g(u_n) \|^2 \tag{3.1}$$

Proof. Let $\bar{u} \in H$ be solution of the problem (1.1). Then

$$<N(T\bar{u}, A\bar{u}), \eta(g(v), g(\bar{u}))> \varphi(g(v), g(\bar{u})) - \varphi(g(\bar{u}), g(\bar{u})) \geq 0, \quad \forall g(v) \in H$$

Since $T: H \to H$ and $A: H \to H$ are η-g-pseudomonotone with respect to the first and second argument of N (.,.), respectively, which implies that

$$<N(Tv, Av), \eta(g(v), g(\bar{u}))> \varphi(g(v), g(\bar{u})) - \varphi(g(\bar{u}), g(\bar{u})) \geq 0, \quad \forall g(v) \in H \tag{3.2}$$

Let $v = u_{n+1}$ in (3.2). We have

$$<\rho N(Tu_{n+1}, Au_{n+1}), \eta(g(u_{n+1}), g(\bar{u}))> \rho\varphi(g(u_{n+1}), g$$

$(\bar{u}))-\rho\varphi(g(\bar{u}),g(\bar{u}))\geq 0$ (3.3)

Let $v=\bar{u}$ in (2.2).We have

$<g(u_{n+1})-g(u_n),g(\bar{u})-g(u_{n+1})>+<\rho N(Tu_{n+1},Au_{n+1}),\eta(g(\bar{u}),g(u_{n+1}))>+\rho\varphi(g(\bar{u}),g(u_{n+1}))-\rho\varphi(g(u_{n+1}),g(u_{n+1}))\geq 0$ (3.4)

Since $\eta(u,v)=-\eta(v,u)\ \forall u,v\in H$, Adding to (3.3) and (3.4), we have

$<g(u_{n+1})-g(u_n),g(\bar{u})-g(u_{n+1})>-\rho[\varphi(g(\bar{u}),g(\bar{u}))-\varphi(g(u_{n+1}),g(\bar{u})-,\varphi(g(\bar{u}),g(u_{n+1}))+\varphi(g(u_{n+1}),g(u_{n+1}))]\geq 0$

i.e. $<g(u_{n+1})-g(u_n),g(\bar{u})-g(u_{n+1})>-\rho[\varphi(g(\bar{u}),g(\bar{u}))-\varphi(g(u_{n+1}),g(\bar{u})-,\varphi(g(\bar{u}),g(u_{n+1}))+\varphi(g(u_{n+1}),g(u_{n+1}))]$

Since $\varphi(.,.):H\times H\to R\cup\{+\infty\}$ is skew-symmetric, we have

$<g(u_{n+1})-g(u_n),g(\bar{u})-g(u_{n+1})>\geq 0$ (3.5)

From (3.5), using Lemma1.1, we obtain

$<g(u_{n+1})-g(u_n),g(\bar{u})-g(u_{n+1})>\frac{1}{2}[\|g(u_n)-g(\bar{u})\|^2-\|g(u_{n+1})-g(u_n)\|^2-\|g(u_{n+1})-g(\bar{u})\|^2\geq 0$

i.e $\|g(u_{n+1})-g(\bar{u})\|^2\leq\|g(u_n)-g(\bar{u})\|^2-\|g(u_{n+1})-g(u_n)\|^2$

Theorem 3.1 Let H be a finite dimensional space and $g:H\to H$ be invertible.If u_{n+1} is the approximate solution obtained from Algorithm 2.1 and $\bar{u}\in H$ is a solution of the problem (1.1),then $\lim_{n\to\infty}u_n=\bar{u}$.

Proof Let be a solution of the problem (1.1).From (3.1),it follows that the sequence $\{\|g(\bar{u})-g(u_n)\|\}$ is nonincreasing and consequently $\{u_n\}$ is bounded. Furthermore,we have

$$\sum_{n=0}^{\infty}\|g(u_{n+1})-g(u_n)\|^2\leq\|g(u_0)-g(\bar{u})\|^2$$

which implies that

$$\lim_{n\to\infty}\|g(u_{n+1})-g(u_n)\|=0 \quad (3.6)$$

Let $\bar{u}$ be the cluster point of $\{u_n\}$.Since $\{u_n\}$ is bounded,there exists a subsequence $\{u_{n_j}\}$ of the sequence $\{u_n\}$ converge to $\bar{u}\in H$.Replacing u_n by u_{n_j} in (2.2) and taking the limit $n_j\to\infty$ and using (3.6), we have

$<N(T\bar{u},A\bar{u}),\eta(g(v),g(\bar{u}))>+\varphi(g(v),g(\bar{u}))-\varphi(g(\bar{u}),g(\bar{u}))\geq 0,\quad\forall g(v)\in H$

which implies that $\bar{u}\in H$ solves the problem (1.1) and

$$\|g(u_{n+1})-g(\bar{u})\|^2\leq\|g(u_n)-g(\bar{u})\|^2$$

Thus it follows from the above inequality that the sequence $\{u_n\}$ has exactly one cluster point $\bar{u}$ and

$$\lim_{n\to\infty}g(u_n)=g(\bar{u})$$

Since g is invertible,so

$$\lim_{n\to\infty}u_n=\bar{u}$$

this completes the proof.

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